Quantum deflation of classical extended objects

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It is shown, that extended particle-like objects should infinitely long collapse into some discontinuous configurations of the same topology, but vanishing mass. Analytic results concerning the general properties and asymptotic rates of such a process are given for 1+1-dimensional soliton models.

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At present, classical particle-like solutions (henceforth solitons) play a significant role in the QFT, since they are considered as reliable models of quantum extended objects, e.g. existing baryons [1-3]. It is widely believed, that if the classical soliton is stable due to either topological or dynamical reasons, then upon quantization by existing techniques its quantum descendant should most likely reveal the same stability properties [1-3,6-9,11-15]. The purpose of this paper is to present arguments, that in the QFT, under rather general circumstances, such a correspondence cannot take place. Namely, we'll describe a peculiar quantum effect, that makes possible a special kind of collapse of such extended objects into certain almost discontinuous configurations of the same topology, but vanishing mass.

Such configurations appear quite naturally within the explicitly lorentz-covariant treatment of the soliton Heisenberg field by means of Newton-Wigner position variables [4]. The latter emerge as the direct result of summing up the pertinent terms in the perturbative solution of eqs. of motion starting from the static classical solution [5] and serve as the covariant collective coordinates of the soliton, which restore the broken Lorentz symmetry and provide the cancellation of corresponding zero modes [6-9]. These covariant coordinates give rise to an effective dynamics, where all the differential operators of the initial theory are replaced by finite differences with the step, proportional to the (effective) Planck constant of the theory $\hbar$ [4,10]. In turn, such effective theory yields a series of c-number copies of classical solution, labeled by an integer N, which are quite analogous to and reveal the same topological properties as the initial soliton, whereas their masses are much less than the classical one and vanish for $\hbar/N \to 0$ [4]. Within such a picture the soliton collapse appears quite naturally as a transition from the classical soliton state into its Nth copy with the same topological charge. The peculiarity of this process

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is, that the dynamics of the collapse can be successfully understood in the quasiclassical approximation, leading to a universal power law for the energy loss into radiation in the asymptotics. Quantum origin of the effect manifests here rather in vanishing energy of the final soliton state and in the model-dependent tunneling effects, which determine the collapse probability $P_{coll} \simeq \exp(-S/\hbar)$, where the value of S can be zero as well.

To avoid unnecessary complications we'll consider these effects within the generic 1+1-dimensional model of a nonlinear scalar field $\varphi(x,t)$, described by the Lagrangean density

$$\mathcal{L}(\varphi) = \tfrac{1}{2}\,(\partial_\mu \varphi)^2 - U(\varphi), \tag{1}$$

which possesses a classical solitary wave solution

$$\varphi_c(x,t) = u\left(\frac{x - vt}{\sqrt{1-v^2}}\right). \tag{2}$$

Soliton quantization with special emphasis on relativistic properties is implemented by means of the exact covariant center-of-mass coordinate [4-9], which in 1+1-dimensions is given by

$$q = \tfrac{1}{2}\,\left(\frac{1}{H}L + L\frac{1}{H}\right), \tag{3}$$

where L is the generator of Lorentz transformations and H is the total Hamiltonian of the system. The operators H, L and the generator of the spatial translations P obey the Lorentz algebra

$$[L,H] = i\hbar P, \quad [L,P] = i\hbar H, \quad [H,P] = 0. \tag{4}$$

The general analysis says, that the c.m.s. coordinate enters the Heisenberg operators always in the combination $x - q(t)$, while the spatial translation is induced by $q \longrightarrow q + a$ [5,7-9]. For our purposes it would be more convenient to introduce q in a dimensionless form by means of the substitution

$$x \longrightarrow \xi(x,t), \tag{5}$$

where

$$\xi(x,t) = H^{1/2}(x - q(t))H^{1/2} = Hx - Pt - L. \tag{6}$$

So in the vicinity of the soliton solution the Heisenberg field $\varphi(x,t)$ is represented as

$$\varphi(x,t) = f(\xi(x,t)) + \Phi(\xi(x,t),t). \tag{7}$$

In eq.(7) we take care of that the operator-valued argument of the soliton field might affect the soliton shape to be different from the classical one and therefore

use new notation $f(\xi)$ for it instead of that in (2). The meson field $\Phi(\xi, t)$, considered as a function of c-number arguments, is taken independent of the generator P of spatial translations of the algebra (4), while the commutation relations between H, L and $\Phi(\xi, t)$ are determined in such a way, that enables to provide the covariance of the whole field. The general procedure of such type has been considered in detail in refs. [6-9]. However, within the present approach we'll deal mainly with the soliton field $f(\xi(x,t))$, which is a covariant operator by construction. Note also, that $\xi(x,t)$ commutes with the mass operator $M^2 = H^2 - P^2$, and so M can be considered as a c-number by dealing with operators of the type $f(\xi(x,t))$.

It can be argued, that the substitution (5-7) represents the general form of the Heisenberg field in the one-soliton sector. Indeed, it was shown in [5], that the replacement (5) appears as a direct result of summing up the pertinent terms in the perturbative solution of equations of motion starting from the single classical soliton. It can be also shown [7-9], that within the expansion in inverse powers of the soliton mass this method is equivalent to the canonical quantization via collective coordinate [11-13]. A quite different relativistic framework for soliton quantization based on the BRST approach has been proposed recently in refs. [14,15]. However, the latter method turns out to be not quite appropriate to study quantum effects considered below, that are essentially non-analytic in $\hbar$ and so lie beyond the quasiclassical perturbation expansion.

The main consequence of the substitution (5) is that in the resulting effective theory all the differential operators of the initial theory are replaced by finite differences of special form. Namely, the dynamical equation, that determines the invariant soliton shape $f(\xi)$, reads [4]

$$f(\xi + i\hbar) + f(\xi - i\hbar) - 2f(\xi) + \frac{\hbar^2}{\mu^2} V'(f(\xi)) = 0, \tag{8}$$

where it is convenient to extract the dimensional (mass) parameter m from the interaction term and to introduce the dimensionless soliton mass

$$U(\varphi) = m^2 V(\varphi), \quad \mu = M/m. \tag{9}$$

Then the eq.(8) is written completely in terms of dimensionless variables. (Recall, that by means of a suitable redefinition of variables [1,4], in 1+1-dimensions we can always treat $\hbar$ as the dimensionless expansion parameter for the soliton sector.)

The mass of the soliton is determined from the following equation

$$\mu^2 = \frac{\mu^2}{2\hbar^2} \int d\xi \; [f(\xi + i\hbar) - f(\xi)][f(\xi - i\hbar) - f(\xi)] + \int d\xi \; V(f(\xi)) \, , \tag{10}$$

where the status of $f(\xi \pm i\hbar)$ is the same as in eq.(8). The detailed discussion of eqs.(8) and (10) is given in ref. [16].

The main consequence of the finite-differences in the dynamical equations is the appearance of what should be called quantum copies of classical solutions [4]. Namely, seeking the solution of eq.(8) in the form of the exponential (Dirichlet) expansion, we obtain the following series of approximate solutions, valid for $\hbar$ sufficiently small, namely

$$f_N(\xi) = u(\alpha_N \xi), \tag{11}$$

where N is an integer,

$$\alpha_N = \frac{1}{\mu_N} + \frac{2\pi N}{\sigma\hbar}, \tag{12}$$

$\mu_N = \mu[f_N]$ is the corresponding dimensionless mass of the field, and σ is equal to the mass of the elementary excitation of the field (meson) divided by m.

For $N = 0$ the singular part in α_N disappears and the corresponding solution $f_0(\xi) = u(\xi/\mu_0)$ survives the limit $\hbar \to 0$. So $f_0(\xi)$ should be considered indeed as the quantum descendant of the classical solution, and so in what follows the case $N = 0$ will be referred to as the quasiclassical one. The masses μ_N of soliton copies are determined from eq.(10) and for small $\hbar$ and $N \neq 0$ are given by

$$\mu_N = \sqrt{\frac{\mu_0}{|\alpha_N|}} \simeq \sqrt{\frac{\hbar}{|N|}} \left(\frac{\mu_0 \sigma}{2\pi}\right)^{1/2}, \tag{13}$$

while μ_0 is the classical soliton mass

$$\mu_0 = \int dx\ u'^2(x). \tag{14}$$

The most essential point here is, that μ_N turn out to be $O(\sqrt{\hbar})$ in magnitude, hence sufficiently smaller, than the classical mass μ_0, but larger, than the typical meson energy, that is of order $O(\hbar)$. Another peculiarity in the result (13) is that μ_N decrease for increasing $|N|$. These properties will be crucial for the soliton collapse.

Now let us consider these and other properties of quantum copies using as an illustrative example the spontaneously broken φ^4-model, that is given by the selfinteraction potential $V(\varphi) = \frac{1}{2}\,(1 - \varphi^2)^2$, and yields the classical kink solution $\varphi_c(x) = \tanh x$. The nonclassical kinks are given by the shape functions

$$f_N(\xi) = \tanh \alpha_N \xi, \tag{15}$$

where $\alpha_N = 1/\mu_N + \pi N/\hbar$. (Recall, that in the φ^4-model with such potential the mass of the elementary meson is $2m$, hence $\sigma = 2$.)

An important property of quantum copies is that for all $N \neq 0$ they carry the same topological charge Q as the classical soliton. For $\hbar/N \to 0 \quad f_N(\xi)$ converge to the limiting shape $f_\infty(\xi)$ in the form of the discontinuous step-like classical solution of the same topology. In the case of an odd classical solution of the φ^4-type, what includes the most important soliton models, we can write without loss of generality

$$f_N(\xi) \to f_\infty(\xi) = \mathrm{sign}\xi \ . \tag{16}$$

According to eq.(13), the mass of $f_\infty(\xi)$ vanishes. This result is essentially non-trivial, since in the purely classical case the kinetic energy of the discontinuous solution like $\mathrm{sign}x$ is proportional to $\int dx\ \delta^2(x)$ and diverges. The reason is, that the energy of quantum copies is determined by the operator fields $f_N(\xi(x,t))$, rather than by the c-number shape functions $f_N(\xi)$. In particular, for sufficiently small $\hbar$ (large N) the nonclassical kinks (15) can be approximated as

$$f_N(\xi) \simeq \tanh\left(\frac{\pi N}{\hbar}\xi\right), \tag{17}$$

hence are $i\hbar$-periodic, and so by substituting them into the differences $[f(\xi \pm i\hbar) - f(\xi)]$ of eq.(10) the latter vanish identically. Thus the quantum origin of soliton copies is crucial for such specific behaviour of their masses.

On account of these properties of quantum copies we should expect, that any given soliton state will be unstable under transition with $\Delta N > 0$, while emitting a corresponding number of mesons. Since the topological charge Q remains unchanged, it should be just the soliton collapse, rather than the soliton decay, that proceeds with $\Delta Q \neq 0$. Now let us present the general analysis of transition from the quasiclassical soliton state into its Nth quantum copy. It would be natural to describe such process in terms of suitable dynamical variables, that continuously transform the initial configuration $f_0(\xi) = u(\alpha_0\xi)$ into the final one $f_N(\xi) = u(\alpha_N\xi)$. The simplest path without infinite potential barriers, that enables such transition, is given by the (linear) superposition of initial and finite soliton states, both taken in the rest frame,

$$f(\xi,t) = c_0(t)f_0(\xi) + c_N(t)f_N(\xi) \ , \tag{18}$$

where the finite energy of the intermediate configurations is provided by the subsidiary condition, imposed on scaling parameters, namely

$$c_0(t) + c_N(t) = 1. \tag{19}$$

So the path (18) reduces to a one-parameter family of trajectories

$$f(\xi,t) = f_0(\xi) + c(t)\Delta f(\xi) \ , \tag{20}$$

where

$$\Delta f(\xi) = f_N(\xi) - f_0(\xi) \tag{21}$$

is a regular function with (exponentially) decreasing asymptotics. The collapsing process is given by the continuous evolution of $c(t)$ subject to boundary conditions

$$c(0) = 0\ ,\quad \dot{c}(t) = 0\ ;\qquad c(\infty) = 1\ ,\quad \dot{c}(\infty) = 0. \tag{22}$$

It would be natural to assume, that the collapse is an adiabatic process (what is confirmed by explicit calculation below). Then the energy loss into radiation will be very low, hence $\Phi(\xi, t)$ will be small compared to $f(\xi, t)$. Granted this, the meson field can be treated perturbatively in the slowly varying c-number soliton background $f(\xi, t)$, while the back reaction of $\Phi(\xi, t)$ on the dynamics of $c(t)$ being neglected. So in the first step we have to consider the proper dynamics of $c(t)$ without radiation. For these purposes we insert the collapsing soliton field (20) into the initial field Hamiltonian and evaluate the energy (mass) of the configuration as an explicit function of $c(t)$ and $\dot{c}(t)$. The result looks like eq.(10) by adding the terms with $\dot{c}(t)$, namely

$$\mu^2 = \tfrac{1}{2}\,\dot{c}^2(t) \int d\xi\ \left[\Delta f(\xi)\right]^2 + \tfrac{1}{2}\,\frac{\partial}{\partial t}\left\{\dot{c}(t) \int d\xi\ \left[\left(1 - \cos\hbar\partial_\xi\right) f(\xi, t)\right]\ \Delta f(\xi)\right\} + \\ + \int d\xi\ \mathcal{H}(\xi, t)\ , \tag{23}$$

where

$$\mathcal{H}(\xi, t) = \frac{\mu^2}{2\hbar^2}\left\{\left[\left(1 - \cos\hbar\partial_\xi\right) f(\xi, t)\right]^2 + \left[\left(\sin\hbar\partial_\xi\right) f(\xi, t)\right]^2\right\} + V(f(\xi, t)). \tag{24}$$

Note, that by integrating over $d\xi$ in eqs.(23) and (24) we can always take the final configuration as the discontinuous $f_\infty(\xi)$. It is definitely correct for transition to $N \to \infty$, but for $\hbar$ small such replacement would be valid actually for any transition from the quasiclassical state into $N \neq 0$.

Calculation of the finite-difference kinetic term T in $\int d\xi\ \mathcal{H}(\xi, t)$ proceeds by means of the following relations [4], valid for any Nth soliton copy

$$\left(\sin\hbar\partial_\xi\right) f_N(\xi) \simeq \frac{\hbar}{\mu_N} u'(\alpha_N \xi), \quad \left(1 - \cos\hbar\partial_\xi\right) f_N(\xi) \simeq \tfrac{1}{2}\,\frac{\hbar^2}{\mu_N^2} u''(\alpha_N \xi)\ , \tag{25}$$

where the prime denotes the derivative with respect to the argument, and the residual in both expressions (25) is $O(\hbar)$. Therefore, for $\hbar$ small the term $\left[\left(1 - \cos\hbar\partial_\xi\right) f_N\right]^2$ can be certainly neglected while compared to $\left[\left(\sin\hbar\partial_\xi\right) f_N\right]^2$. Then on account of the eq.(10) we obtain for the kinetic term of the eq.(24) the following expression

$$\int d\xi\ \left[\left(\sin\hbar\partial_\xi\right) f(\xi, t)\right]^2 = \hbar^2 (c_0^2 + c_N^2) + 2\frac{\hbar^2}{\mu_0 \mu_N}\ c_0 c_N \int d\xi\ u'(\alpha_0 \xi) u'(\alpha_N \xi). \tag{26}$$

The last term in eq.(26) turns out to be $O(\hbar^{5/2})$, hence in $\sqrt{\hbar}$ times less than the first one, and so can be dropped to the lowest order. So the answer for T acquires a very simple form

$$T = \frac{\mu^2}{2}(c_0^2 + c_N^2) = \frac{\mu^2}{2}\left[c^2 + (1-c)^2\right] . \tag{27}$$

Applying the same procedure to the second term in the eq.(23), we can estimate it as $O(\hbar^2)$, and so for an adiabatic process with bounded derivatives of $c(t)$ this term can be neglected as well. So finally the resulting expression for the energy of the collapsing soliton configuration (20) can be written in terms of $c(t)$ and $\dot{c}(t)$ as

$$\mu^2 = \frac{\gamma}{2}\dot{c}^2 + \frac{\mu^2}{2}\left[c^2 + (1-c)^2\right] + W(c) , \tag{28}$$

where

$$\gamma = \int d\xi \ \left[\Delta f(\xi)\right]^2 , \quad W(c) = \int d\xi \ V\left[f_0(\xi) + c\Delta f(\xi)\right] . \tag{29}$$

For the φ^4-model the straightforward evaluation of integrals in eqs.(29) gives

$$\gamma = \frac{8}{3}(2\ln 2 - 1) , \quad W(c) = \frac{4}{3}(c-1)^2\left[\left(8\ln 2 - \frac{16}{3}\right)c^2 + \frac{2}{3}c + \frac{2}{3}\right] . \tag{30}$$

Now let us consider more carefully the most general features of the behaviour of $c(t)$, that are irrespective of the specific properties of the model. Firstly, by definition $W(c=1) = 0$, hence the point $c = 1$ corresponds to the absolute minimum of $W(c)$, while $W(c=0) = \mu_0^2/2$. So the point $c = 1$ will always lye within the classically allowed region for the motion of $c(t)$, whose velocity at this point is $\dot{c}^2 = \mu^2/\gamma$.

Further, with respect to the shape of $W(c)$ on the interval $[0, 1]$ one has to distinguish between two different situations. The first one concerns the case, when both the points $c = 0$ and $c = 1$ belong to the same classically permissible region of motion. Then $c(t)$ will swing nonlinearly between the neighboring turning points, that are given by $c_1 = 0$ and $c_2 > 1$, while the point $c = 1$ will be the intermediate point of the trajectory. Indeed such a situation takes place in the φ^4-model.

The second case corresponds to the situation, when $W(c)$ yields a finite potential barrier between $c = 0$ and $c = 1$. Now in order to reach the final point $c = 1$ while starting from $c = 0$, we have firstly to penetrate the barrier, what gives the quantum-mechanical tunneling amplitude P_{tunn} for the probability of the whole collapsing process. As soon as we penetrate the barrier and so reach the classically permissible region, which the point $c = 1$ belongs to, there starts the process of nonlinear oscillations between the points $0 < c_1 < 1 < c_2$, similar to described above. Since the tunneling proceeds at the energy close to the

initial classical soliton mass μ_0, that is large enough to use the quasiclassical picture, the tunneling amplitude can be estimated through the conventional WKB-approximation

$$P_{tunn} \simeq e^{-S/\hbar}, \tag{31}$$

where S is the corresponding classical action for the forbidden region.

The most strict predictions can be made for the asymptotic behaviour of the collapse, when μ is sufficiently small already and so the oscillations shrink to a small neighborhood of the minimum of $W(c)$, i.e. to the point $c = 1$. Then we can replace the second term in the eq.(28) by $\mu^2/2$, simplifying it up to

$$\frac{\mu^2}{2} = \frac{\gamma}{2}\dot{c}^2 + W(c). \tag{32}$$

Now let us write

$$c(t) = 1 + \tilde{c}(t) \ , \tag{33}$$

where $\tilde{c}(t)$ is small, and represent the collapsing configuration as

$$f(\xi, t) = f_\infty(\xi) + \tilde{c}(t)\Delta f(\xi). \tag{34}$$

Since $V(f_\infty) = V'(f_\infty) = 0$, from the eq.(34) we observe at once, that the expansion of $W(c)$ in powers of $\tilde{c}$ is equivalent to the expansion around the true vacuum value, what gives

$$\mu^2\Big|_{\mu\to 0} = \gamma\left[\dot{\tilde{c}}^2 + \sigma^2\tilde{c}^2\right] \ . \tag{35}$$

As a result, the limiting oscillations will be harmonic with the frequency

$$\nu = \sigma \ . \tag{36}$$

Such peculiar behaviour of the asymptotical dynamics of the system gives rise to one more general feature of the collapsing process. Let us assume, that there exists a set of discrete modes (bound states) $\phi_n(x)$ in the spectrum of meson excitations around the soliton. Then it might seem, that the transition from the initial to finite soliton states might occur through the excitation of certain ϕ_n. However, actually such a picture doesn't take place, since the eigenfrequencies of the discrete spectrum lye always lower, than the meson mass, namely $0 < \omega_n < \sigma$, and so due to the relation (36) the resonance in the meson excitation will be just at the threshold of the continuum. Therefore, the excitation of the bound states will be suppressed compared to the creation of wavepackets, which remove the energy from the soliton. Moreover, it can be argued [16], that the mesonic modes around the final soliton state fall out of the dynamics at all.

To estimate the energy loss into radiation we have to look at the dynamics of the meson component $\Phi(\xi, t)$ during the collapse. We start with the obvious statement, that the energy of the full Heisenberg field $\varphi(x, t)$, that includes both the collapsing soliton and the meson field, certainly conserves. It means, that if the starting point of the collapse is $c = 0,\ \dot{c} = 0,\ \mu = \mu_0$, the total mass of the field remains equal to μ_0 throughout the process. In turn, in the rest frame the wave operator $\Box_{xt}$ acquires in terms of finite differences (after suitable redefinition of the time $t \to mt$) the following form

$$\Box_{xt} \to \partial_t^2 + 2\frac{\mu_0^2}{\hbar^2}\left(\cos \hbar\partial_\xi - 1\right). \tag{37}$$

As a result, the initial field equation, whence the dynamics of $\Phi(\xi, t)$ should have been determined, can be written as

$$\left[\partial_t^2 + 2\frac{\mu_0^2}{\hbar^2}\left(\cos \hbar\partial_\xi - 1\right)\right] \tilde{\varphi}(\xi, t) + V'(\tilde{\varphi}(\xi, t)) = 0\ . \tag{38}$$

In the eq.(38) $\tilde{\varphi}(\xi, t)$ stands for the Heisenberg field transformed to the ξ-reference frame

$$\varphi(x, t) = \tilde{\varphi}(\xi(x, t), t), \tag{39}$$

whereas during the collapse

$$\tilde{\varphi}(\xi, t) = f(\xi, t) + \Phi(\xi, t). \tag{40}$$

The direct consequence of such initial conditions is that $f_0(\xi)$ satisfies the eq.(38) with the required precision. Therefore the region $|c| \ll 1$, where $f(\xi, t)$ is close to $f_0(\xi)$, cannot significantly contribute to the radiation per period of oscillations. The dominant contribution originates from the neighborhood of $c = 1$, where $f(\xi, t) \simeq f_N(\xi, t)$. During the initial stage of the process such a picture will hold, because $f_N(\xi)$ doesn't satisfy the eq.(38), whereas for t large the oscillations of $c(t)$ will shrink to a small neighborhood of $c = 1$. There remains, of course, an intermediate region, where the oscillations won't be harmonic yet, while their amplitude will be large enough for the nonlinear effects to be significant. However, in the asymptotics the radiation will be indeed very small, while at the beginning of the process the nonlinear terms cannot play any significant role, since it takes some definite time for any nonlinearity to develop [17]. Therefore these effects can be essential only for a finite interval of time, and so cannot seriously affect the main properties of the process, while depending to a high degree on the specific features of the model. So in order to retain the generality we'll omit this nonlinear region.

Now let us define more correctly the status of the meson excitations during the collapse. First of all, since the meson field must compensate the c-number

difference in soliton shapes $\Delta f(\xi)$, to the lowest order $\Phi(\xi, t)$ should be treated as a c-number coherent wave. Further, any mesonic excitation should be referred to the initial soliton state with $N = 0$, what we explicitly take account of by putting

$$\Phi(\xi, t) = \tilde{\Phi}(\alpha_0 \xi, t). \tag{41}$$

Making use of relations (25), we can now easily verify, that for such a function the finite-difference wave operator (37) can be replaced by

$$\left[\partial_t^2 + 2\frac{\mu_0^2}{\hbar^2}\left(\cos \hbar\partial_\xi - 1\right)\right] \Phi(\xi, t) \to \Box_{yt} \tilde{\Phi}(y, t), \tag{42}$$

where $y = \alpha_0 \xi$. Actually, the last relation is nothing else, but the manifestation of the quasiclassical nature of the sector with $N = 0$.

After these preliminary remarks we insert the decomposition (40) into the eq.(38) and linearize it with respect to $\Phi(\xi, t)$ in the vicinity of $f_N(\xi)$, while supposing, that the main contribution to $\Phi(\xi, t)$ appears from the neighborhood of $c = 1$. Then we get the simplified equation

$$[\ \Box_{yt} + V''(f_N)\ ]\ \tilde{\Phi} = j, \tag{43}$$

where the radiation generating current j is determined as the residual of the eq.(38), applied to the collapsing soliton field in the vicinity of $f_N(\xi)$

$$-j(\xi, t) = \ddot{c}(t)\Delta f(\xi) + 2\frac{\mu_0^2}{\hbar^2}\left[\left(\cos \hbar\partial_\xi - 1\right) f_N(\xi)\right] + V'[f_N(\xi)]. \tag{44}$$

By means of the relation (25) the difference term in $j(\xi, t)$ can be represented as

$$2\ \frac{\mu_0^2}{\hbar^2}\left[\left(\cos \hbar\partial_\xi - 1\right) f_N(\xi)\right] \simeq -\frac{\mu_0^2}{\mu_N^2}\ u''(\alpha_N \xi). \tag{45}$$

It is easy to verify, that for $\hbar/N \to 0$ we can write

$$\frac{\mu_0^2}{\mu_N^2}\ u''(\alpha_N \xi) \to \frac{Q\mu_0}{\alpha_N}\ \delta'(\xi), \tag{46}$$

where Q is the topological charge and so doesn't depend on $\hbar$. The factor α_N^{-1} in the r.h.s. of the eq.(46) survives the integration over $d\xi$ with the smooth function of the type (41), but vanishes for $\hbar/N \to 0$. So the difference term in the radiation current can be dropped.

Finally, we replace $f_N(\xi)$ in the eqs.(43) and (44) by the limiting $f_\infty(\xi)$, for which $V'(f_\infty) = 0, \quad V''(f_\infty) = \sigma^2$. So in the lowest approximation the eq. of motion for the meson field reads simply

$$\left(\Box_{yt} + \sigma^2\right) \tilde{\Phi}(y, t) = -\ddot{c}(t)\Delta f(\xi). \tag{47}$$

Now, to avoid the excess of precision we retain in $\Delta f(\xi)$ only the first term in the asymptotics for $|\xi| \to \infty$. Then for the case of an odd classical soliton we can represent $\Delta f(\xi)$ explicitly as

$$\Delta f(\xi) \simeq e^{-\sigma|y|}\mathrm{sign}y, \quad y = \alpha_0\xi. \tag{48}$$

Motivated by the same reasons, in the last step we keep only the main harmonics of $c(t)$ with the frequency $\nu = \sigma$, approximating the oscillations by

$$c(t) = 1 + A\cos\nu t. \tag{49}$$

Then the eq.(43) can be easily solved, whence for the radiation energy $\Delta\mathcal{E}$ per the main period of oscillations $\Delta t = 2\pi/\nu$ we find

$$\Delta\mathcal{E} = 4\frac{\left(A\nu^2\right)^2}{\pi}\int\frac{dk}{k^2(k^2+\sigma^2)}\ \sin^2\left[\frac{\Delta t}{2}\sqrt{k^2+\sigma^2}\right] = 3.926\ \frac{A^2\nu}{\pi}. \tag{50}$$

Thus, the radiation energy per period can be estimated as $\Delta\mathcal{E} = \mathrm{Const} \times A^2$, where A is the magnitude of oscillations around $c = 1$. From this relation the asymptotic rate of the collapse can be easily deduced while noticing, that for sufficiently large t the remaining soliton mass is determined through the eq.(35), what gives

$$\mu^2 = \gamma\ \nu^2 A^2. \tag{51}$$

Taking the average over the period, we can write down now the following equation for the energy loss

$$\frac{d\mu}{dt} = -\Delta\mathcal{E} = -\mathrm{Const} \times \mu^2\ , \tag{52}$$

where

$$\mathrm{Const} = \frac{3.926}{\pi\nu\gamma}. \tag{53}$$

Therefore the asymptotical law for the remaining soliton mass $\mu(t)$ reads

$$\mu(t) = \frac{\mathrm{Const}}{t}\ , \tag{54}$$

while the amplitude of oscillations around $c = 1$ and the radiation power decrease as $1/t$ and $1/t^2$ correspondingly.

So the soliton collapse turns out to be indeed a very slow process with the time scale given by the constant (53). It should be stressed, however, that this constant plays here a quite different role compared to the decay time τ of the quantum-mechanical quasistationary state, that decays according to the exponential law $\exp(-t/\tau)$. In the latter case τ determines the time interval,

during which practically all the excitation energy should have been emitted, whereas the soliton collapse reveals a slowly decreasing tail of radiation.

The asymptotic rate of the collapse doesn't depend explicitly on the difference parameter $\hbar$ (or $\hbar/c$, when the speed of light c is restored explicitly). The reason is, that the underlying quantum origin of the effect manifests here in the vanishing energy of the final soliton state, whatafter the whole process can be successfully formulated within the quasiclassical picture. However, there exists another possibility for $\hbar$ to enter the resulting dynamics, namely through the collapse probability P_{coll}. When there are no barriers, as for the φ^4-theory, the whole collapse proceeds in the real time, thence

$$P_{coll} = 1. \tag{55}$$

On the contrary, in presence of a barrier one has firstly to penetrate it, what gives rise of the tunneling amplitude P_{tunn}, and so the collapse probability can be estimated as

$$P_{coll} = P_{tunn} \simeq e^{-S/\hbar} \, , \tag{56}$$

although in both cases the asymptotical behaviour is governed by the same power law (54). So in presence of tunneling the collapse acquires an explicit exponential dependence on $\hbar$, what apparently demonstrates the non-perturbative origin of the effect. Remarkably enough, this time $\hbar$ comes merely from the purely quantum-mechanical considerations, hence it has nothing to do with finite differences and so with the speed of light. So it turns out, that while emerging as an essentially relativistic effect, the collapse actually survives the non-relativistic limit $c \to \infty$.

To conclude let us note, that such a collapse turns out to be an immanent feature of the particle-like classical solutions in the relativistic QFT. The reason is, that the energy density of such extended objects is localized, therefore the covariant c.m.s. coordinate is a well-defined quantity (see [18-20] and references therein). Then the finite-difference structure of the effective dynamics in covariant coordinates implies, that upon quantization the mass of the limiting step-like configuration $f_\infty(\xi)$ will vanish, rather than diverge. In this way the extended particle-like objects induce their collapse by themselves.

Actually, the derived asymptotic law (54) should be valid in more spatial dimensions as well, since the latter maintain the difference structure of the effective dynamics in covariant coordinates [10]. Moreover, the structure of corresponding difference operators coincides with those of the quasipotential approach on the mass shell in momentum space [21], what once more emphasizes the relativistic origin of the effect. So we should expect, that the main features of the effect will survive the transition to more spatial dimensions. The

application of the general framework developed in [10] to this problem will be reported separately.

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